Mathenalchemy
A comic book adventure in math and art.

Hosler & Hosler
This is a comic story set in an amazing world created by mathematicians and artists. Mathemalchemy is a multi-media art installation designed, fabricated and constructed by a team of 24 artists and mathematicians, celebrating the beauty, joy and creativity of mathematics in all its forms. The Mathemalchemists worked on the installation mostly during the two calendar years 2020 and ‘21, when they were all constrained by pandemic restrictions on meeting and travel. In January 2022, the installation started its career as a traveling exhibit. First ports of call were the National Academy of Sciences in Washington DC (Jan-June 2022) and Juniata College (June-December 2022), followed by Boston University (January-March 2023) and University of British Columbia (April-October 2023); URL mathemalchemy.org/events/#upcoming will continue to provide updated information on further tour stops and associated events.

Meet the Mathemalchemists!

Top Row: Emily Baker, Bronna Butler, Edmund Harriss, Elizabeth Paley, Kimberly Roth, Edward Vogel, Dominique Ehrmann, Susan Goldstine;  
Middle Row: Dorothy Buck, Rochy Flint, Li-Mei Lim, Kathy Peterson, Henry Seegerman, Jake Wildstrom, Vernelle A. A. Noel, Tasha Pruitt;  
Bottom Row: Ingrid Daubechies, Faye Goldman, Sabetta Matsumoto, Samantha Pezzimenti, Jessica K. Sklar, Mary William, Daina Taimina, Carolyn Yackel.

Mathemalchemy was supported by these generous sponsors!
Up past a square root of -1, through the Cantor set, and right along a transcendental path, there is a world of wonders nestled in an infinite orchard of Sierpinski trees.

It's a strange realm where math and magic swirl together in a great nexus of possibilities.

See the glow in those windows? Someone is playing with those possibilities right now.
YES, DEAR?

ARE YOU REALLY GONNA MAKE THIS BIG MAP COME ALIVE?

IN A WAY, I SUPPOSE.

THIS MAP HAS BEEN IN OUR FAMILY FOR CENTURIES.

OUR ANCESTORS BELIEVED IT WAS A FLAT REPRESENTATION OF A COMPLETELY DIFFERENT THREE DIMENSIONAL WORLD CALLED MATHEMALCHEMY.
"THREE DIMENSIONAL?"

LIKE LENGTH, DEPTH, AND WIDTH?

THOSE DIMENSIONS?

"YES!"

OUR ANCESTORS BELIEVED THIS MAP WAS A SPECIAL PLACE WHERE THE THREE DIMENSIONS OF OUR WORLD...

...AND THE THREE DIMENSIONS OF MATH EMALCHEMY...

...OVERLAP!

BUT IT'S NOT A COMPLETE OVERLAP?

VERY GOOD. THE FLAT MAP ONLY SHOWS TWO OF THEIR DIMENSIONS. I'M LOOKING FOR THE EQUATION TO UNLOCK THEIR THIRD DIMENSION.
IS IT WORKING? NO

MY MATH IS CORRECT BUT I CAN'T FIGURE OUT WHAT THIS ONE SYMBOL MEANS.

SURE.

CAN I SEE?

IT KIND LOOKS LIKE THE MUSICAL NOTE HIGH C IS BEING SQUISHED BY A WARPED MUSICAL STAFF.

HMM.

MAYBE WE JUST NEED TO PLAY THAT NOTE DURING THE MATHEMAGICAL SPELL.

DON'T BE SILLY, THAT'S NOT HOW--

TOOOOT!

POOP

EMMY!
uh oh.

HMM.

THAT BIRD COULDN'T FLY VERY WELL.

PERHAPS IT WAS A PENGUIN.

I BEG YOUR PARDON?

THAT WAS NO PENGUIN, SIR.
Well, whatever it is, it’s stuck in our net.

I hope it hasn’t damaged any of the knots we’ve caught.

Help me.

It’s tangled in a trefoil knot.

Please.

You’re a slimey fish. Can’t you just wriggle out?

I’m not a fish!

Please, this thing is really tight. Can’t you just untie me?

Impossible.

A trefoil is a non-trivial mathematical knot.

It has no loose ends. It can’t be untied.

What?

But...

...you were in the water.

We saw you.
Will you explain it?

Sure.

Imagine my neck popped off, thusly.

Aah!

Now, I wind myself like this.

Through here.

And then I fuse the two ends.

Trefoil knot

No loose ends, see?

A trefoil knot can't be untied because it's one continuous thing.

And this is just the simplest of knots.

There are much more complex knots in our net.

None of which can be untied, I'm afraid.

That's ok...
I made some loose ends.

SKEOOW! She chewed it apart!

TSK! It's not a knot now!

Don't freak out, I'll fix it.

How's this?

You've completely changed the species.

We can't study that!

Why not?

That's an unknot!

It's literally a big zero!

I... I'm sorry. I don't know what's going on.

Just stand over there and don't break anything else.

Mom?
MOM!
YOU THINK THE SKY IS YOUR MOM?

WHAT KIND OF CREATURE ARE YOU?

ISN'T IT OBVIOUS, HERON?

SHE'S A MATHEMATICIAN.

FINISHED? ALMOST.

AHHHHHHHH

HUFF PUFF OKAY... ALL DONE.

Cough...

GOOD.
SO, THE SKY SHADOW LOOMING OVER MATHEMALCHEMY IS... YOUR MOTHER?

YES! SHE’S DOING THE CALCULATIONS TO FIGURE OUT HOW TO ENTER THIS WORLD.

SEE THE PAGES SWIRLING FROM HER HANDS?

INDER WE CALL HER THE MATHEMATICIAN.

THERE’S A SHADOW OF ANOTHER, SMALLER MATHEMATICIAN SITTING ON A ROCK FORMATION WE CALL "STACK-OF-BOOKS."

IT’S PLAYING ONE OF THESE.

O-KAY.

MY FLUTE!

IT FLOATED DOWN AND LANDED ON ME A FEW MOMENTS AGO.

WHEN I SURFACED TO INVESTIGATE, THE SHADOW ON "STACK-OF-BOOKS" WAS GONE. THIS SEEMS TO HAVE HAPPENED RIGHT AROUND THE TIME YOUR FLUTE APPEARED.

I PLAYED A NOTE DURING MY MOM’S CALCULATIONS AND THEN JUST POPPED INTO THIS PLACE.
MOM! I'M HERE! SHE CAN'T HEAR YOU. SHE'S JUST A FLAT SHADOW.

HOW AM I SUPPOSED TO GET HOME?

I DON'T KNOW, BUT, HARRIET AT CONWAY'S CURIO SHOP MIGHT.

THEY HAVE ALL SORTS OF MATH-Y STUFF THAT MIGHT BE USEFUL.

I DON'T HAVE ANY MONEY.

THEN YOU'LL NEED SOMETHING TO TRADE.

PERHAPS A JOHNSON SOLID?

MY MOM SAYS A JOHNSON SOLID IS A CONVEX, THREE-DIMENSIONAL SHAPE IN WHICH ALL OF THE FACES ARE REGULAR POLYGONS.

OKAY

I HAVE NO IDEA WHAT ANY OF THOSE WORDS MEAN.

YEAH, MY MOM HAD TO EXPLAIN IT TO ME IN TWO PARTS.
First, this square pyramid is a convex three-dimensional shape.

In a convex three-dimensional shape, a line drawn between points on any two separate faces will be inside the shape.

As opposed to what?

Something like this stellated octahedron, I guess. See? You can connect points on some faces and the line you get is outside the shape.

So, lines between faces gotta be inside. Got it.

The second bit of my mom’s explanation is that each face of a Johnson solid is a regular polygon.
A polygon is a shape with at least three straight sides and three angles.

But in a **regular** polygon all the angles are equal and all sides are equal length.

So, this square pyramid is a Johnson solid because each face is a regular polygon...

...and, any line I draw between points on different faces will be inside the shape.

Nice. My mom would be proud of you.

She taught you well.

Is she a mathematician?

No, she's an artist, so she knows a lot about dimensions and shapes and stuff.

Right, now, she's completely flat and living in a math mural she's painting in town.
Should I take this one to trade?

Maybe, but let's look at a few more.

Whoa!

There's a lot.

These are only a few.

There are almost 100 Johnson solids.

Should I just take all of them?

No, that wouldn't be good for the health of the Knotical Bay ecosystem.

You gotta pick.

But... which one is best?

My mom says mathematics and art are both beautiful, but that their beauty lies in the eye of the beholder.

...and I'm the beholder.

Okay, I pick...
THAT ONE!

OOO! YOU'VE PICKED THE ELONGATED SQUARE GYROBICUPOLA. GOOD CHOICE!

PUT IT IN THIS BAG. IT WILL BE EASIER TO CARRY.

THIS ONE IS UNIQUE AMONG ALL JOHNSON SOLIDS.

SWEET.

THANKS.
I can put my flute in here, too.

So, can you introduce me to Harriet?

I'll get you to the shop, but I can't go in.

Octopuses make herantsy.

How come?

Well, she's very particular about the items in her store.

The problem is octopuses are really curious and we have a lot of arms.

We love fiddling with all of the trinkets, but she's always worried we'll break stuff or take something.

Did you ever?

Of course not, but when I was little I did get stuck in one of her Klein bottles.

It took me forever to get out.

Whoa. Is that mathematicalchemy?

Yep.
PRETTY COOL, HUH?
Thanks for all of the help, I'm Emmy, by the way.

Whoop, what are you doing?

The Curio Shop isn't open yet.

I'm gonna drop you off somewhere scenic.

Gulp... It's pretty high up...

...and there's no guardrail...

This is the lighthouse, honored guest. How can we help you on your quest?

Hey, Del and Nabla, would you show my friend Emmy how to get to the Curio Shop? She's a mathematician.

We'll tell the way you must traverse in couplets of our rhyming verse.

Oh! That's...uh...very clever.
THE WAY TO GO IS DOWN AND ROUND UNTIL YOU WALK UPON THE GROUND.

SO, I JUST WALK DOWN THIS RAMP TO GET TO THE CURIO SHOP?

AT THE GROUND’S VICINITY, THEN TURN TO FACE INFINITY!

uh...

...THAT’S POETIC, BUT NOT VERY SPECIFIC.

IF PRECISE DIRECTIONS ARE WHAT YOU SEEK, YOU SHOULD SPEAK TO BROTHER ZEKE.

AWK!

OKAY, CAN YOU TELL ME THE WAY, ZEKE?

C(1)=[[1+20i]/[2i+1]]^{16}/\zeta
h: [0, 1] \rightarrow \mathbb{R}^3
\{<C(\cos(10\pi t), \sin(10\pi t), 10+20t)>, t \in [0, \pi/6]>
\{<2 \cos(10\pi t)-3, -2 \sin(10\pi t), 0>, t \in [\pi/6, \pi/3]>
\{<3+10, 12+10(\cos(2\pi t)), 0>, t \in [\pi/3, \pi/2]>

OH, WOW. THAT IS PRECISE. I KNOW JUST THE PATH TO GET THERE NOW. THANKS!
Our words can make us poetic and concise, but sometimes we need formulae to be precise.

Hey, that's Cayley's mom.

Octo Pi's mural is full of math but hidden behind this tower path.

And just like that we've reached the ground, have a good time looking around.

Thanks, puffins!

Okay, according to Zeke's equation, I need to turn right.
Excuse me, I'm looking for Harriet.

That's me, dear.

Oh, hi. I'm Emmy.

Yes, the mathematician from the stack of books. News travels quickly around here.

Come in, come in.

Thanks.

Cayley said you might know how I can get home.

I brought something to exchange for your help.

Ooo, that is a very nice elongated square gyrobicupola.

It is unique among Johnson solids, you know.

That's what Cayley said, but he didn't explain.

Ah, allow me then.
When you zoom in on any corner of an elongated square gyrobicupola (ESG), it looks the same as any other ESG corner.

This characteristic is very similar to uniform solids like the cube. When you zoom in on any corner of a cube, it looks the same as any other corner.

But when you zoom out from the corner of a cube, you always get the same view, no matter what corner you’re on.

This is where the ESG is different from the uniform solids.

Although all of the corners look the same close up...

...when you zoom out from an ESG corner, the view you get can be very different depending on what corner you’re on.

No other solid we know of has these two properties. It is unique and beautiful.

So, it’s a good trade? You’ll help me?
Gracious me, I would have helped you without asking for payment, but this is a very nice egg and I am more than happy to add it to my collection.

It's all yours.

Tess the tortoise is who you want to talk to. She's been walking Xeno's path forever, and she's learned many of the secrets and mysteries of mathemalchemy.

She might be on the path right now.

That's awesome.

How do I get there?

Xeno's path runs right past the store. You can't miss her. She's the only tortoise on the island.

Thank you so much, I... uh...

What's wrong?

The bag with my flute is gone.

I put it down on the floor right beside a cup.

Ugh... oh.
Was it a brown, clay cup?

Yeah.

Oh, dear.

I suspect Proteus the mouse has your flute.

There was a mouse in that cup?

No, the mouse was the cup. Proteus is a shape-shifter and, frankly, a bit naughty sometimes.

Toot!

That's it!

You better hurry. He's quick!

Gasp!

Excuse me.

Excuse me.

Excuse me.

It's the mathematician!

Sound the trumpets!

Hey!

Ho o o o o o o o o o o o o o o o
Bow! Bow before the mighty mathematician!

Huh? No, don’t...

We are honored that you have joined our humble festivities, oh queen of quotients!

I’m not really...

Let the sieving of the primes begin!

The what now?
You may start when ready, supreme solver of sums.

Yeah, I could, but...

...maybe you should explain this to everyone just for the...uh...new squirrels.

Excellent idea, oh Titan of teaching.

Today, as is our tradition, we shall find all of the prime numbers between 1 and 100.

I'm not stalling for time or anything, but...

Maybe you should take a minute to explain prime numbers, too.

A prime number is a whole number greater than 1 whose only factor are 1 and itself!

Now we are ready to begin.

Right.

Wait! Wait! Wait!

Eliminate the 1 for her. An exalted mathematician can't be bothered with eliminating 1.

Oh, thanks.

Now you may begin, most hallowed cypher of sums.

Great.
NO! NO! NO! STOP!

Begging your pardon, but surely we should remove all of the multiples of 2? Such a simple exercise is beneath your Stupendous skills.

OK?

Remove the multiples of 2!

So, why did we do that?

Why? Surely you know, great mathematician?

Of course, but I am... uh... testing you.

Oh! Forgive my impertinence!

Two is prime because it's only factors are 1 and 2, but every multiple of 2 (like 4 and 6) has at least three factors: 1, 2, and themselves. So they are eliminated because they are not prime.
OH, I THINK I GET IT. DO THE NEXT ONE.

BUT... I AM NOT WORTHY.

SURE YOU ARE.

THIS IS SO CLEVER. EACH TIME YOU REMOVE THE MULTIPLES, YOU ENSURE THAT THE NEXT REMAINING NUMBER IS PRIME. 4 WAS REMOVED WHEN WE GOT RID OF THE MULTIPLES OF 2, SO NEXT UP IS 5. BETTER REMOVE THOSE MULTIPLES NOW.

AS YOU WISH, OH PRODIGIOUS PROBLEM SOLVER.

AND IF WE JUST DO THIS OVER AND OVER UNTIL WE GET TO 100, THE ONLY NUMBERS LEFT WILL BE PRIME.

YES, OH SUPREME SIEVER.

REMOVE THE MULTIPLES OF 3!

WHAT WAS THAT? NO ONE SAID TO SOUND THE TRUMPETS.

THAT WAS MY FLUTE.
I gotta go. You're in charge.

Me? I can't.

You already have.

You're a natural mathematician.

But...

Good luck.

I'm a mathematician.

Excuse me!

Comin' through!

Toodoodoot

I hear you, Proteus.

Whoa.
THERE'S SO MUCH MORE TO EXPLORE

I HEAR YOU, YOU RAT!

WILL YOU STOP PLAYING?

IT SOUNDS HORRIBLE!

PLEASE SHUT.
Oop.

Oh, dear.
Does it really sound that bad?

Um... that's my flute, ma'am.
Yes, it is.
Are you Tess?
The Only Tortoise on the island.
Don't be too mad at Proteus, I asked him to bring you to me. And he chose a rather... indirect method.
You were looking for me?

I assumed you would like to know the way home.

You're not the first mathematician to pop into Mathe Malchemy.

How did you know that?

They spend so much time trying to get here that they forget to plan for the return trip.
How do you know the way back to my home?

That's an interesting story. Climb on and I'll tell you.

Every day I walk here on Zeno's path. When I start, I walk half the length of the path and then stop to rest for a bit.

When I'm ready, I walk half of the remaining distance, then I stop and rest again.

Do you know how long it takes me to get to the end of the path if I do that over and over?

A day?

Forever.

Infinity.

I will never reach the end of Zeno's path going half the distance each time like that.

There is always a little bit of distance left to cut in half.

I don't understand.

Um... Tess?
How can you walk "infinity" every day?

Well, I never actually make it to the end of the path, do I?

But the thing is, each time I walk the path, I meditate and brush up against the infinite.

I imagine the possibilities.

When it's time for tea, I turn around and walk home like normal.

When I think about the uncountable worlds and dimensions full of mathematician's plucking away at the fabric of reality, it is hardly a surprise when a few of them are successful and break through.

Their arrival here, like yours, was inevitable.

So, I needed to be ready. I have thought and studied and planned as I walked Zeno's path.
So, what's the secret? What do I need to do?

Imagine. Imagine the walls we build between math and art, between science and story.

Now imagine those walls crumbling.

Those are the barriers that hold you back. When they fall in your mind's eye, you will be able to move freely between your home and mathemalchemy.

Thanks, Tess. Can I come back and explore some more with my mom?

Of course. Infinity always has room for one more.

Okay, then. Imagine.

Attention! Attention everyone! We need to get ready!
VISITORS ARE ON THEIR WAY.
You’ve read the story, now visit the exhibit!
Explore the world of Mathemalchemy from anywhere in the world by visiting mathemalchemy.org. Read about how the exhibit was created, explore the mathematical concepts woven into the art, or read a little more about your favorite characters. Use the QR code above to get started now!

About the Cartoonists

**Jay Hosler** is a professor of biology at Juniata College. He is also a cartoonist who has written and drawn several comics and graphic novels about science. You can learn more about his books and read tons of science comics at jayhosler.com. He's also on Instagram at @hoslerjay

**Maxwell Hosler** has a degree in Mathematics from the College of Wooster. He has spent his life trying to explain math to his father. This comic is evidence that some of that work has paid off.

Thanks, Juniata!
We are profoundly grateful to the Juniata College Department of Biology, Department of Mathematics, and the Office of the Provost for providing the financial support required to print this comic. Juniata is a super cool place to work and an even better place to get a degree. Stop by for a visit if you’re ever in the area or visit our website to learn more. [www.juniata.edu](http://www.juniata.edu).

The story and art in Mathemalchemy: A comic book adventure in math and art are (c) Jay Hosler and Maxwell Hosler.